**Discrete Mathematical Structures**

**Week-6**

**Long Descriptive Questions**

**1. Show that ¬p ʌ (¬q ʌ r)) v (q ʌ r) v (p ʌ r) => r**

¬p ʌ (¬q ʌ r)) v (q ʌ r) v (p ʌ r) is conditionally equivalent to r, the truth value of the two statements is the same for all possible truth values of the variables, assuming that r is true.

Simplifying **LHS -** distributive law of propositional logic

¬p ʌ (¬q ʌ r)) v (q ʌ r) v (p ʌ r) = (¬p ˅ q ˅ r) ∧ (p ˅ r)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **¬p ˅ q ˅ r** | **p ˅ r** | **r** |
| T | T | T | T | T | T |
| T | T | F | T | T | F |
| T | F | T | T | T | T |
| T | F | F | T | T | F |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | F | F | F |

In the truth table, we assume that r is true, and we evaluate the truth values of the two statements for all possible combinations of truth values of p, q, and r.

We can see that the truth values of the two statements are the same for all possible truth values of the variables, assuming that r is true. Therefore, we can conclude that ¬p ʌ (¬q ʌ r)) v (q ʌ r) v (p ʌ r) is conditionally equivalent to r

**2, Show that p → q, p → r, q → ¬r and p are inconsistent**

To show that p → q, p → r, q → ¬r, and p are inconsistent, we need to demonstrate that it is not possible for all of them to be true at the same time.

First, we can use the premises p → q and p → r to conclude that q and r must both be true if p is true.

Then, using the premise q → ¬r, we can conclude that ¬r must also be true if q is true.

However, we have already concluded that q and r must both be true if p is true, which means that ¬r cannot be true if p is true.

Therefore, it is not possible for all the premises to be true at the same time, and they are inconsistent.

To summarize, we can show that p → q, p → r, q → ¬r, and p are inconsistent by demonstrating that they lead to a logical contradiction. Specifically, if p is true, then q and r must both be true, but q → ¬r implies that ¬r must also be true if q is true, which contradicts the earlier conclusion that r must be true if p is true. Therefore, the premises are inconsistent and cannot all be true at the same time.